WASHINGTON STATE
WORKFORCE TRAINING AND EDUCATION COORDINATING BOARD
MEETING NO. 124
JANUARY 31, 2008

HIGH SCHOOL GRADUATION REQUIREMENTS: CAREER AND TECHNICAL EDUCATION AND THE THIRD CREDIT OF MATH

At the November meeting, the Workforce Training and Education Coordinating Board (Workforce Board) discussed the third credit of math that should be required for high school graduation. In 2007, the Legislature had passed 2SHB 1906 directing the State Board of Education (SBE) to amend high school graduation requirements to include a minimum of three credits of mathematics, one of which may be a career and technical course equivalent in mathematics, and prescribe the required content. The Board discussed a motion recommended by staff that would have offered advice to the SBE regarding that requirement (Attachment A). There was a motion to table discussion, and that motion passed. Consequently, no formal recommendation form the Workforce Board was presented to the SBE.

Since November, there have been several developments. The State Board for Community and Technical Colleges (SBCTC) met and discussed the same issue and at a special meeting adopted a resolution offering its recommendation to the SBE (Attachment B). The Office of Superintendent of Public Instruction (OSPI) issued a preliminary draft paper proposing new content standards for math during high school (excerpted in Attachment C). The standards are categorized into six areas: (1) algebraic structure and number systems; (2) functions and analysis; (3) geometry and geometric measurement; (4) data, statistics, and probability; (5) communication; and (6) reason and problem solving.

On January 9 and 10, 2008, the SBE met and discussed the requirement for the third credit of math. The SBE directed its staff to prepare a draft rule that would establish certain requirements (please see the first page in this tab after the recommended motion). The SBE indicated that it would discuss the draft rule at its March 26-27, 2008, meeting, and would plan on adopting a rule at its May 14-15, 2008 meeting.

The SBE's direction to its staff for the draft rule has two parts. Under the first part, students would be required to complete a third credit of math in Algebra II that meets the content standards identified by OSPI. Career and Technical Education course work could satisfy this requirement if the course of study had comparable content to Algebra II. Under the second part, a student, with the approval of his or her high school counselor and parent/guardian, could elect to replace the Algebra II requirement with a third math credit that furthers an approved program of study that leads to a specific career goal.

In addition to the attachments mentioned above, this tab contains a recommended motion and a discussion of the arguments for and against requiring Algebra II for high school graduation.

Board Action Requested: Adoption of the Recommended Motion.

RECOMMENDED MOTION

WHEREAS, The 2007 Legislature passed 2SHB 1906 which directs the State Board of Education to amend high school graduation requirements to include a minimum of three credits of mathematics, one of which may be a career and technical course equivalent in mathematics, and prescribe the required content;

WHEREAS, The State Board of Education had directed its staff to prepare a rule by which students will have two options for fulfilling the third credit in math:

1) A student could complete a third credit of math in Algebra II that meets the new math content standards identified by the Office of Superintendent of Public Instruction, and Career and Technical Education course work could satisfy this requirement if the course of study had comparable content to Algebra II.

2) A student, with the approval of his or her high school counselor and parent/guardian, could elect to replace the Algebra II requirement with a third math credit that furthers an approved program of study that leads to a specific career goal.

WHEREAS, The Workforce Training and Education Coordinating Board has a statutory responsibility to advocate for meeting the needs of students, workers, and employers for jobs that do not require a baccalaureate degree;

WHEREAS, The Workforce Training and Education Coordinating Board supports high school education requirements that recognize the importance of education that is personalized to individual student career and education goals, including access to career and technical education; and,

WHEREAS, In order to graduate from high school students must complete a high school and beyond plan that includes the student's goal for the year after high school graduation and the course work during high school that will enable the student to achieve that goal.

NOW THEREFORE BE IT RESOLVED, That the Workforce Training and Education Coordinating Board supports the direction of the State Board of Education on January 10, 2008 regarding the third credit of math required for high school graduation.

State Board of Education Request of Staff for a Draft Rule on the Third Credit of Math
January 10, 2008

The Board of Education directed staff to draft rule language for review at the March State Board of Education meeting that requires all students to complete a third credit of math in an Algebra II course that aligns with the new math standards and meets the content standards to be approved by the Board. This course requirement can be completed through an approved career and technical education course of study that is comparable in course content but allows the student to earn more than one credit to complete. This will be in effect for the Class of 2013. This decision aligns with the Board's specified purpose of the high school diploma—to prepare all students for success in postsecondary education, gainful employment and citizenship.

Upon completion of a second credit of mathematics that meets the 9^{th} and 10^{th} grade level expectations, students may elect to pursue, or continue to pursue, an approved program of study that leads to a specific career goal. This election shall allow the student to replace the Algebra II requirement with a third math credit that furthers this approved program of study. The election shall require approval by a high school counselor or administrator, and shall include a counseling session with the student and family/guardian that at a minimum makes sure everyone understands the future opportunities that may be unavailable to the student by making this choice. It shall also encourage the student to take additional math courses during the remainder of their high school studies that assist them towards their career goals and maintain their math skills.

Arguments For and Against Requiring the Completion of Algebra II In Order to Graduate from High School

Arguments For Requiring Algebra II or its Equivalent[1]

Shortage of Workers in Science, Information Technology, and Engineering

Workforce projections suggest a growing shortage of U.S. citizens having the kinds of technical skills that build on such courses as Algebra II. (Committee on Science, Engineering, and Public Policy, 2007)

Preparation for college programs, including family-wage workforce programs

Most community and technical college professional-technical programs leading to family-wage jobs require Algebra II for entry or completion. (Washington State Board for Community & Technical Colleges, 2007)

Algebra II is a prerequisite for College Algebra, the mathematics course most commonly required for postsecondary degrees. Virtually all college students who have not taken Algebra II in high school will need to take remedial mathematics. (ACT, 2006) The strongest predictor for baccalaureate degree completion is the level of math course completed in high school. (Adelman, 1999, 2006)

Completion of Algebra II is required by the Higher Education Coordinating Board for admission to Washington four-year colleges and universities.

Employment and education data show that Algebra II is a "threshold course" for high-paying jobs. In particular, five in six young people in the top quarter of the income distribution have completed Algebra II. (Carnevale & Desrochers, 2003)

Equity

Students most likely to opt out of algebra when it is not required are those whose parents are least engaged in their children's education. The result is an education system that magnifies inequities and perpetuates socioeconomic differences from one generation to the next. (Haycock, 2007) This has a disproportionate impact on students of color.

The best available evidence is limited but suggests that raising math standards has at most a negligible negative impact on dropout rates and at best can improve graduation rates by improving the quality of course offerings available to students, especially if couple with significant support for students making the transition into high school math. (Lee & Burkam, 2003; Bishop & Mane, 2004)

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[1] Appreciation goes to Bill Moore of the State Board for Community and Technical Colleges for his assistance in preparing this section.

Cost of Remediation

In 2005, $17.2 million was spent to provide 3,110 FTE of pre-college course work to recent high school graduates. (SBCTC Research Report no. 06-5)

Other States' Requirements

Thirty-five states have or are slated to increase math requirements within the next few years. (Linda Plattner, State Board of Education presentation January, 2008)

Arguments Against Requiring Algebra II or its Equivalent

Algebra II is Not Required for Most Jobs

Very few individuals use Algebra II on their job. Fewer than 5 percent of jobs require the use of Algebra II. (Carnevale & Desrochers, 2003)

Even many well-paid jobs are filled by individuals who never completed Algebra II. Carnevale and Desrochers report that 47 percent of all those in well-paid jobs did not complete Algebra II and 63 percent of well-paid blue-collar workers did not complete Algebra II. Carnevale and Desrochers conclude that, "Geometry is the benchmark-level course for students who are ultimately employed in well-paid skilled blue-collar jobs… ." Carnevale and Desrochers also note that, "Test scores in nonacademic, occupational-focused areas showed a relationship with hourly wages similar to that of more academic knowledge."

The Shortage of Workers in STEM Fields has Other Causes

Recent studies from Georgetown University and the Urban Institute, Rand Corporation, Harvard and Stanford Universities tell us that the numbers of students who are educated in science and engineering fields has held steady over time; however, the numbers of graduates entering employment in those fields are declining. They've also determined that more than enough students are taking and passing Algebra II across the country to fill every available college level STEM seat; however, there is no increase in the numbers choosing to do so.

Not Required for Postsecondary Opportunities

There are four major postsecondary options that do not require Algebra II: certificated workforce education programs at community and technical colleges, many apprenticeship programs, private career schools, and the military. Getting to the "Tipping Point" of one year of postsecondary training plus a credential does not require ever completing Algebra II. The only postsecondary option that may be precluded by not completing Algebra II in high school is immediate entry into a four-year institution. Completing Algebra II by itself, however, does not enable immediate entry into a four-year institution. Immediate entry also requires the completion of two years of foreign language, and most students who are admitted to a four-year university in Washington have a grade point average substantially above a 3.0.

Students who do not meet the requirements for immediate entry into a four-year intuition after high school may still enter a four-year institution by first attending a community college and then transferring. Even if remedial math is taken at a community college, beginning college in non-credit bearing math courses has little effect on graduation rates (55 percent vs. 60 percent). (Adelman, 1998)

Adelman shows a correlation between completing higher levels of math in high school and completing a baccalaureate degree. Adelman's research, however, does not control for the academic ability of students. In other words, it is probably the case that more academically adept and goal oriented students take higher levels of math in high school; these same students would be more likely to complete a baccalaureate degree even if they had not taken higher math in high school.

Negative Impact on Student Access to Career and Technical Education

Out of over a thousand Career and Technical Education (CTE) courses in Washington, only a handful of courses incorporate the full range of concepts of Algebra II. Since CTE is an elective, requiring Algebra II would likely lead schools to **decrease their CTE offerings** in order to make room for Algebra II course taking. This would be easier for the schools to do than to revamp their CTE courses to incorporate Algebra II.

Fix Math Instruction Before Considering Requiring More of the Same

There is a general consensus that the way math is taught is not working for many kids. We should first change the curriculum and the teaching methods and not add more of the same that is not working.

Recent work of the National Science Foundation suggests that we need to start students much earlier in the process. That we are doing a disservice to young people by waiting until high school to begin using algebra in the classroom, and should think about beginning lessons in elementary school that build steadily so that children are prepared and enthused to tackle high school level algebra and beyond.

Personalized Education

The Governor's Committee on Education Reform and Funding (GCERF), the blue ribbon committee that lead to standards-based reform in Washington, recommended the common standards be set at the 10th grade level, recognizing what students learn after the 10th grade should not be the same for all students. The idea was that education should be personalized, with programs of study after the 10th grade that would vary depending on the student's future career goals.

Graduation Rates

The 70 percent rate at which ninth graders graduate on-time with their class is arguably the most important problem in Washington K-12 education. There is little research on the impact of requiring Algebra II on graduation rates. About half of Washington students do not pass the 10th grade math WASL set at the Algebra I level, and as a result, the legislature just delayed the Algebra I level WASL as a graduation requirement.

Shortage of Math Teachers

There is already a severe shortage of math teachers.

Most States do Not Require Algebra II

Only 15 states require Algebra II for high school graduation. (Achieve.org)

ATTACHMENT A

November 8, 2007 Recommended Motion Tabled by the Board

WHEREAS, The 2007 Legislature passed 2SHB 1906 which directs the State Board of Education to amend high school graduation requirements to include a minimum of three credits of mathematics, one of which may be a career and technical course equivalent in mathematics, and prescribe the required content;

WHEREAS, The State Board of Education (SBE) is currently studying Career and Technical Education (CTE) and its relationship to math competencies; and

WHEREAS, CTE program standards established by OSPI require all CTE coursework to include application and contextualization of the related Essential Academic Learning Requirements and Grade Level Expectations (EALRs and GLEs), including skills needed to meet state assessments; and

WHEREAS, Math-enhanced CTE coursework will aid learning by providing students the opportunity to apply math knowledge and content within a context that is relevant to their interests; and

WHEREAS, The examination of CTE coursework that is rich in academic content will be conducted by an advisory committee established under ESSB 6023, passed by the 2007 Legislature;

NOW THEREFORE BE IT RESOLVED, That the Workforce Training and Education Coordinating Board urges the State Board of Education to establish a career and technical education option for meeting the graduation requirements for a third credit of math, as set forth in 2SHB 1906, and to consider career and technical education courses that enhance students' competency in the application of algebraic and geometric concepts as providing sufficient rigor for that requirement; and

BE IT FURTHER RESOLVED, That the Workforce Training and Education Coordinating Board also urges the State Board of Education to consider the recommendations of the advisory committee, established under ESSB 6023, in determining the quantity of math content needed in order for career and technical education courses to be deemed an equivalent credit for the third credit math requirement.

ATTACHMENT B

STATE BOARD FOR COMMUNITY AND TECHNICAL COLLEGES
RESOLUTION 07-12-41

A resolution relating to proposed definitions of the mathematics credits required for high school graduation.

WHEREAS, the 2007 Legislature passed 2SHB-1906 which directs the State Board of Education to amend high school graduation requirements to include a minimum of three credits of mathematics, one of which may be a career and technical course equivalent in mathematics, and describe the required content; and

WHEREAS, the State Board for Community and Technical Colleges continues to address issues of students success through the Transition Math Project in an effort to reduce remedial math course-taking among recent high school graduates entering community and technical colleges; and

WHEREAS, community and technical college academic programs, high-wage workforce programs, and most medium-wage workforce programs require Algebra II skills for entry or completion; and

WHEREAS, the State Board for Community and Technical Colleges strongly supports a diversity of pathways and options for students entering colleges in the system, including opportunities to learn mathematics in applied and career-related contexts;

THEREFORE BE IT RESOLVED that the State Board for Community and Technical Colleges recommends to the State Board of Education that the third credit of math to be required for high school graduation be aligned with Algebra II-level concepts and skills, and for high school career and technical programs develop courses which include practical math applications reflecting those concepts and skills.

APPROVED AND ADOPTED on December 19, 2007.

SUPERINTENDENT OF PUBLIC INSTRUCTION

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December 3, 2007

To Interested Educators, Policy Leaders, and Citizens of Washington:

This letter is my opportunity to introduce the **preliminary draft** of the proposed revision of the Washington K-12 Mathematics Standards. The **final version** of the revised mathematics standards is scheduled to be delivered to the Legislature on January 31, 2008.

This preliminary draft of the revised mathematics standards is our initial response to the State Board of Education's (SBE) *Washington State Mathematics Standards Review and Recommendations* report. The Board's report includes several basic recommendations, including increasing rigor and clarity, focusing attention on important ideas at each grade (For example, providing more guidance about the specific performance levels of computational fluency and use of algorithms.), balancing mathematical processes and content, and improving the usability of the standards document for a variety of audiences. The full SBE report, which includes the rationale for each of the recommendations, can be downloaded from the SBE web site at http://www.sbe.wa.gov/documents/WAFinalRecommendationsMath8-30.pdf.

This preliminary draft of the revised mathematics standards was developed by a group of experts from Washington State and across the nation between early October and December 4, 2007. Teachers, mathematicians, mathematics educators, curriculum specialists, and business/community leaders participated in this very intensive process. A list of all people who have participated is included with the draft on the project web site. I am very grateful for the tremendous effort put forth by these people to craft a document that we are now able to share with the citizens of Washington State for their review and comment.

As you read this first draft, remember that standards are the **goals** for mathematics learning. Standards indicate what all students are supposed to learn, typically in a grade-by-grade format. Standards do not specify either how teachers should teach or what instructional materials should be used in instruction. Decisions on curriculum materials and on the pedagogy to use can be made only after there is agreement on the goals for instruction. We have done our best to address issues of computational fluency, thoughtful use of algorithms, increased rigor, and focus on critical ideas at each grade.

You will also notice that the organizational structure of the preliminary draft is different than that of the organizational structure of the current Washington K-10 Mathematics Standards. In the revised document, at each grade in K-8, and for Grades 9-12 as a whole, there are three or four "content priorities," two "process priorities," and at most grades a collection of lesser important content that supports these priorities. For each content and process idea, you will find a list of "student expectations" that provides the detail about what students should know and be able to do.

In addition, at Grades 9-12, we intend to develop course descriptions for both a traditional sequence (Algebra I, Geometry, Algebra II) and an integrated sequence (Integrated Math 1, Integrated Math 2, Integrated Math 3). However, we have not yet had time to complete drafts of these course descriptions that we are ready to release for public comment. This work is underway, but we need to have feedback about the appropriateness of the standards for Grades 9-12 before we can complete the course descriptions. Washington's graduation testing requirement is now set at the 10^{th} grade. These Grade 9-12 draft standards do not attempt to delineate a "graduation" benchmark. Washington educators and policymakers will need to make that decision.

I hope that you will read the draft carefully and consider how well it could provide guidance for teachers about the mathematics that every student in Washington State should learn. We are very interested in obtaining feedback from a wide variety of citizens of the state so that we can improve this draft as we move toward the January 31 deadline. For this purpose, an evaluation form is available on the website where this preliminary draft is posted at http://www.utdanacenter.org/wamathrevision/. We need your feedback as quickly as possible so that we can remain on schedule for delivery of the final version to the Legislature on January 31.

During December and January, there will be a series of focus groups and information sessions at which citizens can make comments in person. In addition, we expect to receive many comments electronically through the web site and via emails sent to **wastandards@austin.utexas.edu.**

Finally, as you read this document, remember that it is only a first draft. It is not the finished product. The people who produced this draft will continue to refine it, while also considering all of the input that we receive. We hope that you share our excitement about the progress we have made in improving the standards as well as our optimism that the revised standards will be the first step in improving the mathematics learning of all students in Washington State.

Sincerely,

Terry Bergeson

Dr. Terry Bergeson
State Superintendent
Of Public Instruction

Washington State Mathematics Standards

Preliminary Draft
December 4, 2007

Please offer suggestions by December 30, 2007, on how this draft can be improved.
Send suggestions to wastandards@austin.utexas.edu,
or visit www.k12.wa.us for more feedback options.

Background and Rationale

In 2007, the Washington Legislature amended the state's Basic Education Act with a clear statement about preparing every student for full participation in a global society:

> The goal of the basic education act for the schools of the state of Washington . . . shall be to provide students with the opportunity to become responsible and respectful global citizens, to contribute to their economic well-being and that of their families and communities, to explore and understand diverse perspectives, to enjoy productive and satisfying lives, and to develop a public school system that focuses on the educational achievement of all students, which includes high expectations for and prepares students to achieve personal and academic success . . .

The Act further calls for students to know and apply core concepts and principles of mathematics, think analytically, logically, and creatively, and reason and solve problems. More specifically, the Washington Legislature took several bold steps to promote improved mathematics teaching and learning in the state.

This 2007-2008 revision of the Washington Mathematics Standards has been organized in response to a mandate from the Legislature, as part of a two-step process. The first step involved a review by the State Board of Education (SBE) of the current mathematics standards, and the second step called for a revision of the standards by the Office of the Superintendent of Public Instruction (OSPI).

As part of the growing attention to mathematics at the state level, the State Board of Education also created a Math Panel, which commissioned a review of the current standards by the firm Strategic Teaching, led by Linda Plattner. The final review was approved by the SBE as a report, *Washington State Mathematics Standards Review and Recommendations*, which can be downloaded from the SBE website.

The report highlights seven major recommendations (quoted from the Executive Summary):

1. Set higher expectations for Washington's students by fortifying content and increasing rigor.
2. Make clear the importance of all aspects of mathematics: mathematics content, including the standard algorithms; the conceptual understanding of the content; and the application of mathematical processes within the content.
3. Identify those topics that should be taught for extended periods at each grade level and better show how topics develop over grade levels.
4. Increase the clarity, specificity, and measurability of the Grade Level Expectations.
5. Write Essential Academic Learning Requirements that restructure the standards document to clarify grade-level content priorities and that reflect both the conceptual and procedural sides of mathematics.
6. Create a standards document that is easily used by most people.
7. Create small, expert Standards Revision Teams for each grade band and systematically collect feedback on the revised standards.

As soon as the report was approved as official recommendations of the State Board of Education, OSPI issued a Request for Proposals for an organization to manage the standards revision process. From the proposals submitted, the proposal written by the Charles A. Dana Center at the University of Texas at Austin was chosen.

These draft K-12 mathematics standards have been developed by a team of Washington educators, mathematics faculty, and citizens with support from staff of the Office of the Superintendent of Public Instruction and invited national consultants, and facilitated by staff of the Dana Center. The individuals who have played key roles in this project are listed below:

Washington Educators and Community Leaders
Dana Anderson, Stanwood-Camano School District
Tim Bartlett, Granite Falls School District
Millie Brezinski, Nine Mile Falls School District
Jane Broom, Microsoft
Jewel Brumley, Yakima School District
Bob Dean, Evergreen Schools
John Burke, Gonzaga University
Shannon Edwards, Chief Leschi School
Angela English, Arlington School District
John Firkins, Gonzaga University (retired)
Russ Gordon, Whitman College
Katherine Hansen, Bethel School District
Tricia Hukee, Sumner School District
Michael Janski, Cascade School District
Russ Killingsworth, Seattle Pacific University
James King, University of Washington
Art Mabbott, Seattle Schools
Kristen Maxwell, Educational Service District 105
Rosalyn O'Donnell, Ellensburg School District
M. Cary Painter, Chehalis School District
Patrick Paris, Tacoma School District
Tom Robinson, Lake Chelan School District
Terry Rose, Everett School District
Allen Senear, Seattle Schools
Lorna Spear, Spokane Schools
David Thielk, Central Kitsap School District
Johnnie Tucker, retired teacher
Kimberly Vincent, Washington State University
Virginia Warfield, University of Washington
Sharon Young, Seattle Pacific University

Dana Center Facilitators
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Cathy Seeley
Susan Hudson Hull

National Consultants
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Angela Andrews, National Louis University
Diane Briars, Pittsburgh Schools (retired)
Cathy Brown, Oregon Department of Education (retired)
Dinah Chancellor, Consultant
Philip Daro, Consultant
Bill Hopkins, Dana Center
Barbara King, Dana Center
Kurt Krieth, University of California at Davis
Bonnie McNemar, Consultant
David D. Molina, Consultant
Susan Eddins, Illinois Math and Science Academy (retired)
Wade Ellis, West Valley College, CA (retired)
Margaret Myers, The University of Texas at Austin
Lynn Raith, Pittsburgh Schools (retired)
Jane Schielack, Texas A&M University
Carmen Whitman, Consultant

OSPI Mathematics Personnel: Observers and Project Support
George W. Bright, Special Assistant to the Superintendent
Barbara Chamberlain, Interim Director
Larry Davison, Math Helping Corps Administrator
Ron Donovan, Teaching & Learning
Dorian "Boo" Drury, Teaching & Learning
Lynda Eich, Assessment
Karen Hall, Assessment
Robert Hodgman, Assessment
Mary Holmberg, Assessment
Anton Jackson, Assessment
Karrin Lewis, Teaching & Learning
Jessica Vavrus, Teaching and Learning Operations and Programs Administrator

Introduction to the Preliminary Draft of the Proposed Mathematics Standards

This document presents a proposed draft of revised standards for K-12 mathematics in Washington schools. The writers have attempted to include various identified strengths of the current Washington standards (Essential Academic Learning Requirements and Grade Level Expectations), including, but not limited to, a strong commitment to mathematical thinking, reasoning, and problem solving, as well as strong content development in certain areas, particularly in the development of algebraic thinking in grades K-8. Furthermore, the writers have looked carefully at recognized standards from high-performing states and countries and have considered the changing needs of the workplace and higher education, in order to raise the rigor of the mathematics expected so that every student is challenged to achieve the highest level possible.

This draft document is intended to stimulate discussion, with the hope that extensive feedback will be offered by Washington educators and other citizens to continue to improve it before its final adoption in early 2008. The writers involved with the development of this draft reiterate the fundamental goal asserted by the legislature in Washington's Basic Education Act—that every single student in Washington deserves, needs, and will receive a high-quality education suitable for preparing a citizen for full participation in a global society. These mathematics standards are presented in the spirit that they will be taught to and achieved by every Washington student.

The standards will be useful in actualizing this goal only with the help of educators and citizens throughout the state. Please send your feedback to wastandards@austin.utexas.edu, or visit OSPI's website (www.k12.wa.us) for information on other ways to submit feedback. Please send feedback no later than December 30, 2007. This short timeline is necessary in order to meet legislative requirements.

Essential Academic Learning Requirements

The mathematics standards described here build on Washington's longstanding commitment to teaching mathematics content and mathematical thinking. In particular, the Essential Academic Learning Requirements (EALRs) and Grade Level Expectations (GLEs) provide a foundation upon which to build a new era of defining standards based on identifying a few priorities at each grade level within a well-balanced mathematics program. The current EALRs, listed below, represent threads in the development of mathematical content, problem solving, reasoning, and communication that should be obvious in these Preliminary Draft Mathematics Standards.

EALR 1: The student understands and applies the concepts and procedures of mathematics.

EALR 2: The student uses mathematics to define and solve problems.

EALR 3: The student uses mathematical reasoning.

EALR 4: The student communicates knowledge and understanding in both everyday and mathematical language.

EALR 5: The student understands how mathematical ideas connect within mathematics, to other subject areas, and to real-life situations.

However, as noted in the SBE recommendations, the revised standards were to incorporate a structure that more clearly delineated grade-level priorities, leading to a new format for the standards, described below.

Format of Proposed K-8 Mathematics Standards

As identified in the SBE's adopted recommendations, the current standards structure has been modified to more clearly represent what is expected at each grade level and, perhaps more importantly, what is most essential at each grade level. The format for each grade level from kindergarten through grade 8 shows three to four mathematics content priorities, each of which consists of a connected set of knowledge and skills. Each grade level also includes a process priority on Reasoning/Problem Solving and one on Communication. Within each of these content and process priorities, a set of student expectations is described on the left side of a two-column format. The right column has been set aside for examples and clarifying notes, although the expectations themselves have been written to clearly communicate what is expected without further clarifications or examples. Mathematics standards for grades 9-12 are described according to the same type of priorities and expectations and organized in mathematical content threads, as described below in the section about the proposed high school standards.

The paragraphs describing the priorities at each grade level for K-8 and for the grades 9-12 threads represent a critical and unique element of this revision. They describe the nature of the most important mathematics to be learned and show how the individual expectations fit together within and across grade levels to build a cohesive body of mathematical tools and mathematical thinking.

The expectations themselves describe the particular conceptual knowledge and procedural skills expected to be taught and learned. They also describe how students are expected to apply the mathematics they are learning to reason, think, analyze, and solve problems of increasing sophistication as they move from one grade to the next.

In addition to the paragraph descriptions of grade-level priorities and the specific student expectations, additional mathematics content is described for each grade level under the heading Supporting Ideas. These Supporting Ideas similarly include a paragraph description and specific statements of student expectations.

The format used for these standards addresses priority concerns and recommendations from the SBE's recommendations, including clearly identifying priorities at each grade level and communicating a balance between mathematical content and mathematical processes. It also addresses the first step in making the standards clearer and more usable by communicating to a broad general audience, especially through the paragraphs. Educators can also use the paragraphs to understand where most classroom time should be spent and to clarify how a cluster of knowledge and skills fits together. Educators can then use the expectations and related examples and clarifications to pinpoint specific skills and content they are expected to teach. Additional formats for other purposes may be developed in the future.

An important distinction is that these standards specify the *priority* concepts, skills, and processes that every student is to know and be able to do. They do not specify the complete curriculum for all students. For example, students interested in careers in mathematics-related fields will most likely complete more than the described three years of high school mathematics, and, consequently, will learn content beyond these standards.

In addition, some content may be valuable for students to see in the classroom, because it helps them better understand core content and/or make connections among ideas, without all students being expected to master it. Such content might not be in the standards, but might well be in the implemented curriculum. The key question is whether these draft standards describe the priority content that every student should know and be able to do.

Please note that, excerpted from the current mathematics EALR/GLE publication, the same glossary of mathematics words and selected terms used in the standards is included as Appendix A. As part of the process of offering feedback, please suggest any additional terms to be considered for inclusion in this glossary.

Format of Proposed Grades 9-12 Mathematics Standards

These proposed revised high school standards are organized by content "threads" with additional attention paid to mathematical processes. The four threads are

- Algebraic Structure and Number Systems
- Functions and Analysis
- Geometry and Geometric Measurement
- Data, Statistics, and Probability

As in grades K-8, the two processes are Reasoning/Problem Solving and Communication.

In addition to describing two threads on Geometry and Statistics/Data Analysis, the high school standards separate the domain of algebra into two additional threads—1) Algebraic Structure and Number Systems and 2) Functions and Analysis—to emphasize the different aspects of this domain. The Algebraic Structure and Number Systems thread addresses the extension of K-8 work on number to number systems and their properties and generalizes arithmetic to computing with variables, expressions, and equations. The Functions and Analysis thread focuses on the relationships among quantities and the use of functions to represent and model problem situations, make predictions, analyze and interpret relationships, and generalize the behaviors of these relationships. These two threads also foreshadow the distinction between algebra and analysis in higher mathematics.

Each of the threads is organized into several Priorities, and under each Priority there is a set of student expectations. For each of the two process priorities there is also a set of student expectations.

The intent of this organization is to outline standards that would address the content in the first three years of a high school mathematics sequence—either a three-year integrated mathematics program or a more traditional course structure organized around Algebra I, Geometry, and

Algebra II—without making decisions at this time as to how the standards would be allocated among three courses.

In the near future, these standards will be organized in two ways. One will be to describe standards for each course in an Algebra I, Geometry, Algebra II sequence, while the other will be to describe standards for an Integrated Mathematics 1, Integrated Mathematics 2, Integrated Mathematics 3 sequence. Because neither of these structures is privileged over the other for this initial draft, high schools organized in either direction can make use of the standards.

In Washington, there is a mandate to increase the high school mathematics graduation requirement to three years. At this time, however, much discussion is taking place as to what particular courses or content might comprise those three years of mathematics. The writers of this document do not assume that the three years of content included in the grades 9-12 standards here will necessarily represent the mathematics that will eventually be expected of all students. That decision remains for others. However, the three years of mathematics content outlined here are designed to prepare all students for the needs they will face after high school, either in entering the workforce or in pursuing additional training or education.

Mathematical Processes K-12

At each grade level, two Priorities are described related to mathematical processes—Reasoning/Problem Solving and Communication. This choice was made for two reasons. First, it is sometimes difficult to distinguish between reasoning and problem solving, since reasoning is typically required as a part of solving problems. It seems appropriate for organizing the standards, as well as for the ultimate quality of student learning, to acknowledge the close connection between reasoning and problem solving. Second, this specification of mathematical processes matches recommendations in the SBE's *Washington State Mathematics Standards Review and Recommendations* report.

Each grade level includes specific process priorities, with mathematics content embedded within the priority description and within related expectations. Furthermore, mathematical processes are embedded within many of the content expectations, where the content calls for such connections. While it is impossible to separate processes and content completely, the explicit description of processes at each grade level calls attention to their importance within a well-balanced mathematics program. Attention to mathematical processes is one of the identified strengths of the current mathematics standards described in the State Board of Education's report.

Although there is much common language used to describe the mathematical processes across the grades, it is important to recognize that the content for problem solving as well as the language and symbolism used in communication of understanding become more sophisticated across grades. These changes reflect increasing complexity of content and increasing rigor as students deal with this increasing complexity, much in the same way as reading skills develop from grade to grade with increasingly complex reading material. Making sense of the standards at any grade requires simultaneous examination of the content priorities and the process priorities for that grade.

Technology

It is impossible to specify the variety of technologies that might help students learn mathematics. New technologies are constantly being developed, and older technologies are being improved. No standards document can foresee all of the kinds of technologies that might be available within a few years. However, mathematics standards should take into account the possibility that technology can help students learn mathematics and address more complex mathematics than might otherwise be possible.

Occasionally these proposed revised mathematics standards indicate when technology should be used or should not be used. In most cases, however, the decision is left up to teachers as to when technology might be appropriate and how students should use it. The standards describe the mathematics students need to learn, not necessarily all of the tools that teachers might use to help students reach those goals. Teachers need to be knowledgeable about how technology can support students in learning mathematics, not replace their learning. More importantly, teachers need to know what mathematics students need to understand so that the students can not only use the technology, but also can make sense of the outputs from that technology.

A helpful discussion of the appropriate use of technology can be found in Washington's College-Readiness Standards produced as part of the Transition Math Project.

A Note About "Understanding"

The SBE's recommendations include a concern regarding the use of the word "understand" in the current standards. The writers have taken care to avoid this word in specific student expectations, as it can often be difficult to measure and difficult to identify when a student truly understands a mathematical idea.

But avoiding the words "understand" or "understanding" can eliminate important parts of the teaching and learning process in mathematics, as described by the mathematician Jim Lewis:

> [W]ords like "understand" have taken on an undesirable meaning to some and we thus have made the shift to wanting standards expressed in a very active language ("The student represents and uses . . .", "The student is expected to . . ."). As such, there appears to be no place for a standard where the teacher is to define and the students are to understand the rational numbers and further understand (and be able to work with) fractions, decimals and percents as three ways that we represent rational numbers.
>
> Dr. W. James Lewis, University of Nebraska, personal correspondence, November 2007

The writers of these proposed standards, therefore, have chosen to selectively allow for the word "understand" or "understanding" to appear in some of the paragraphs that describe grade-level priorities.

What Next?

The mathematics standards that will finally be adopted by the Washington Legislature represent an important step in revamping mathematics teaching and learning in the state. The standards will be a critical foundation, but only the first step. The success of the mathematics standards rests in related implementation efforts that will likely include attention to

- instructional materials,
- large-scale and classroom assessment,
- teacher and administrator professional development,
- graduation requirements and related state and local policies, and
- local district, school and classroom decisions on how the standards will be implemented.

Such system-wide efforts take time, always balancing the need to "raise the bar" with the reality of what is possible in real schools with real teachers and real students. This is a time for discussion, engagement, and concrete suggestions on how to support teachers, students and communities in reaching ambitious and exciting goals in mathematics.

Please send your feedback to **wastandards@austin.utexas.edu**, or visit OSPI's website **(www.k12.wa.us)** for information on other ways to submit feedback. **Please send feedback no later than December 30, 2007.**

High School Priority A1: NUMBERS, EXPRESSIONS, AND OPERATIONS
(Algebraic Structure and Number Systems).
Students need variables and expressions to solve problems and analyze a wide range of situations. They generate expressions and rewrite them as equivalent expressions for different purposes. Students extend arithmetic operations to radicals, complex numbers, polynomials, and rational algebraic expressions.

Performance Expectations	Explanatory Comments and Examples
Students are expected to:	
A.1.a Compare and order real numbers expressed as fractions, decimals, square roots, or in scientific notation and determine their relative positions on the number line.	
A.1.b Select either appropriate approximations or exact representations of real numbers, depending on the context, and justify the selection.	In a real-world situation, like carpentry or engineering, often an answer is expressed as a decimal approximation with varying degrees of precision. But in strictly mathematical contexts, exact answers are often preferred.
A.1.c Evaluate polynomial and algebraic expressions, including expressions containing radicals, absolute values, and rational exponents.	
A.1.d Explain how the whole, integer, rational, real, and complex number systems are related to each other, and identify the number system(s) under which a given algebraic equation can be solved.	Examples: *Under which number systems can each be solved?* • $x^2 = 1$ • $x^2 = \frac{1}{4}$ • $x^2 = 2$ • $x^2 = -2$

A.1.e Describe the possible real values that a variable can assume in an algebraic expression based on given conditions.	Examples: • *For what values of a would 1/a be an integer?* • *For what values of a would -a be positive?* • *For what values of a would* $\sqrt{5-a}$ *be a real number?*
A.1.f Rewrite a numeric or algebraic expression that contains radicals or rational exponents, and convert between radical notation and exponential notation.	Examples: • $\sqrt[5]{16}=2^{\frac{4}{5}}$ • $\sqrt[3]{24}=2\sqrt[3]{3}$ • $\frac{2^{-2}3^{2}5}{2^{2}3^{-3}5^{2}}=\frac{3^{5}}{2^{4}5}=\frac{243}{16\bullet 5}=\frac{243}{80}$ • $\sqrt{x^{2}+1}=(x^{2}+1)^{\frac{1}{2}}$ • $\frac{x^{2}}{\sqrt{x}}=x^{\frac{3}{2}}$ • $\frac{\sqrt{x}}{x^{2}}=x^{-\frac{3}{2}}=\frac{1}{x^{\frac{3}{2}}}=\frac{1}{\sqrt{x^{3}}}=\frac{1}{x\sqrt{x}}$ • $\frac{a^{-2}b^{2}c}{a^{2}b^{-3}c^{2}}=\frac{b^{5}}{a^{4}c}$
A.1.g Rewrite a polynomial expression using algebraic rules, including factoring and combining like terms.	This includes • using the distributive property, • factoring a monomial term from a polynomial, • factoring quadratic trinomials with integer coefficients that have factors with integral coefficients, and • factoring special quadratic trinomials (e.g., difference of two squares)
A.1.h Rewrite a rational algebraic expression whose denominators are polynomials of degree no greater than two, and use conjugates to rationalize denominators of the form $x\pm\sqrt{a}$.	

A.1.i Add, subtract, and multiply polynomials, and divide polynomials by polynomials of degree one.	
A.1.j Add, subtract, multiply, divide, and simplify rational algebraic expressions.	Operations on rational expressions exclude those containing radicals.
A.1.k Add, subtract, and multiply complex numbers, and divide complex numbers by real numbers.	

High School Priority A2: EQUATIONS AND INEQUALITIES

(Algebraic Structure and Number Systems)

Students solve equations and inequalities that arise in a wide range of problems and applications. Students choose algebraic representations and tools that are appropriate to a problem situation, interpret the meaning of solutions, and explain the limitations of those solutions.

Performance Expectations	Explanatory Comments and Examples
Students are expected to:	
A.2.a Identify the values for the variable(s) that make a given equation or inequality true.	Example: *For what values of the variable(s) are the following true?* • $3x + 2 = 7$? • $3x + 2 = y$? • $x + 2x = 3x$? • $\lvert 6 \rvert < \frac{1}{x}$
A.2.b Solve linear equations and inequalities.	
A.2.c Rewrite equations and inequalities containing the absolute value of a linear expression as compound equations or inequalities, and solve.	
A.2.d Use systems of two linear equations or inequalities to represent problem situations; solve the systems using algebraic, graphic, or numeric methods; and interpret the solution(s) in the context of the problem.	• Solving systems of linear inequalities should focus on graphic solution methods. • Systems should be extended to other systems (e.g., linear and quadratic, two quadratics, etc.), but note that not all students are accountable for this in three years of high school mathematics.

A.2.e	Solve a literal equation for a given variable.	Example: *Solve $A=p+prt$ for p*
A.2.f	Solve quadratic equations with integer coefficients over real or complex numbers using factoring, graphing, completing the square, and/or the quadratic formula.	This is an algebraic skill applied in F.4.a.
A.2.g	Solve equations over the real or complex numbers involving a variable under the radical, identify extraneous solutions, and explain how they arose.	Example: *Solve* • $\sqrt{5x-6}=x$ • $\sqrt{5x-6}=2$
A.2.h	Solve equations containing rational algebraic expressions of the form $\frac{f(x)}{g(x)}=k$, where *f(x)* and *g(x)* are polynomials of degree no greater than two.	Example: *Solve* $\frac{x^2+6x+1}{x-1}=2$
A.2.i	Simplify exponential equations using the properties of exponents and logarithms, and evaluate using technology.	Students can solve using logarithms or graphs.

High School Priority F1: FORMALIZING FUNCTIONS
(Functions and Analysis)
Students formalize and deepen their understanding of functions, including their defining characteristics and uses. Students apply this understanding to functions whose domains are discrete, non-discrete, and continuous.

Performance Expectations	Explanatory Comments and Examples
Students are expected to:	
F.1.a Distinguish between relations and functions, and determine whether a given relation is also a function.	
F.1.b Translate among representations of functions expressed in various forms, including algebraic rules (*y*= and *f(x)* forms), tables, graphs, and verbal descriptions.	The various forms include • an algebraic rule that assigns a unique value to each element of the domain, • a table that exhibits such a correspondence explicitly, • a graph in the coordinate plane, and • a verbal description. When translating from a table or graph to an algebraic rule, to avoid ambiguity, students will need to know certain conditions, for example, that a function is quadratic.
F.1.c Determine the value of a function at particular points when the function is expressed in various forms, including algebraic rules, tables, graphs, and verbal descriptions.	
F.1.d Determine the domain and range of a function, the effect that a change in domain would have on the range, and the limitations on the domain and range in specific contexts.	Functions include polynomial, radical, rational, piecewise (absolute value, step, and piecewise defined), and exponential.
F.1.e Describe the shape of the graph of a function based on the underlying algebraic rule and key characteristics about the algebraic rule based on the shape of the graph.	

F.1.f Solve a problem by examining the problem, choosing an appropriate function to model the situation, justifying the selection, determining the limitations on the domain and range, and determining the reasonableness of the solution.	
F.1.g Perform transformations of the form $f(x-a)$, $f(x)+b$, $f(cx)$, $df(x)$ or combinations of these on a function $f(x)$.	Example: *The graph of* $f(x) = -5(x-3)^2 + 2$ *is a transformation of the parabola* $f(x)$ *that is reflected over the x-axis, shifted 3 units to the right and 2 units up, and expanded vertically by a factor of 5.*

High School Priority F2: LINEAR FUNCTIONS
(Functions and Analysis)
Students use linear functions, and functions containing an absolute value of a linear expression, to represent and model problem situations, make predictions, and analyze relationships. Students use their understanding of linear functions to answer questions and solve problems that involve a constant rate of change.

Performance Expectations	Explanatory Comments and Examples
Students are expected to:	
F.2.a Simplify linear expressions and solve linear equations and inequalities to answer questions about situations modeled by linear functions.	Given a function describing a situation, answer questions (mathematical or contextual) about specific x and y values, i.e., given x, find y, and given y, find x. When solving an equation, it can be thought of as two functions, $f(x)$ and $g(x)$, where $f(x) = g(x)$. The solution to the equation is the solution to this system.
F.2.b Describe how changes in the parameters of linear functions and functions containing an absolute value of a linear expression affect their graphs and the relationships that they represent.	In the case of linear functions, the word parameter refers to m, slope, and b, the constant term. Examples: • *If the slope changes from 3 to 5, how will the graph of the function change?* • *If the rate of savings changes from \$300 to \$500 dollars per month, how will the graph of the function change?*
F.2.c Sketch the graph of a line expressed in standard, point-slope, and slope-intercept forms.	Standard form: $Ax + By = C$ Slope intercept form: $y = mx + b$ Point-slope form: $(y - y_{1)} = m(x - x_1)$

F.2.d	Analyze and translate among linear functions expressed in various forms (with an algebraic rule, a table, a graph, and a verbal description) and identify the slope and y-intercept in each representation.	
F.2.e	Express arithmetic sequences in both explicit linear and recursive function forms, translate between the two, and explain how the rate of change is represented in each form.	Example: *Identify arithmetic sequences with linear functions whose domain consists of whole numbers, and express arithmetic sequences of the form* $y(n) = a + bn$ *as recursive relations of the form* $y(n+1) = y(n) + b$ *with* $y(0) = a$*, and vice versa.* "Explicit" form is often referred to as "closed" form.
F.2.f	Write an algebraic rule for a linear function given two points on its graph, or its slope and a point on its graph.	The related concept is that two parameters are sufficient to determine a line.
F.2.g	Generate and rewrite linear equations in standard, slope-intercept, and point-slope form as is most appropriate for a particular situation.	Rewriting is of the form: • Standard to slope-intercept, and vice versa. • Point-slope to either standard or slope-intercept.

High School Priority F3: EXPONENTIAL FUNCTIONS

(Functions and Analysis)

Students build upon their understanding of linear functions that have a constant rate of change to exponential functions that have a proportional rate of change. Students use exponential functions to represent and model situations, make predictions, and analyze relationships.

Performance Expectations	Explanatory Comments and Examples
Students are expected to:	
F.3.a Simplify exponential expressions and solve exponential equations to answer questions about situations modeled by exponential functions.	
F.3.b Sketch the graph of an exponential function of the form $y=ab^x$ and describe the effects that changes in the parameters a and b will have on the graph.	
F.3.c Analyze and compare the proportional rate of change in exponential functions to the constant rate of change in linear functions.	
F.3.d Analyze and translate among exponential functions expressed in various forms (with an algebraic rule, a table, a graph, and a verbal description), and, from each representation, identify the intercept, the asymptote, and whether it is an increasing or decreasing function.	Pay attention to asymptotes and behavior when the base is close to 1. Decay and growth are significant applications. The intent is to compare the behavior of exponential functions to that of linear and other functions, e.g., quadratic.

F.3.e	Express geometric sequences in both explicit exponential and recursive function forms, translate between the two, and explain how rate of change is represented in each form.	Example: *Identify geometric sequences with exponential functions whose domain consists of whole numbers, and express geometric sequences of the form* $y(n) = ab^n$ *as recursive relations of the form* $y(n+1) = y(n) \cdot b$ *with* $y(0) = a$, *and vice versa.*

High School Priority F4: QUADRATIC AND OTHER FUNCTIONS
(Functions and Analysis)
Students use quadratic and radical functions to represent and model situations, make predictions, and analyze relationships. Students extend their study of functions to include functions containing polynomials of higher degree and rational algebraic expressions.

Performance Expectations	Explanatory Comments and Examples
Students are expected to:	
F.4.a Simplify quadratic expressions and solve quadratic equations and inequalities to answer questions about situations modeled by quadratic functions.	Given a function describing a situation, answer questions (mathematical or contextual) about specific x and y values, i.e., given x, find y, and given y, find x. Pay special attention to extraneous solutions because this is the first time that students will encounter them.
F.4.b Sketch the graph of a quadratic function, and describe the relationship between the characteristics of the graph and • a and c in $f(x) = ax^2 + bx + c$; • a, h and k in $f(x) = a(x-h)^2 + k$; and • r, s and a in $f(x) = a(x-r)(x-s)$.	
F.4.c Analyze and translate among various forms of a quadratic function (algebraic rule, table, graph).	
F.4.d Graph inverse functions of the form $f(x) = \frac{a}{x} + b$, and specify the asymptotes of the graph.	
F.4.e Sketch the graph of rational functions of degree 1 or 2, specify the limitations on the domain and range, and determine important characteristics of their graphs including asymptotes, intercepts, and holes.	

F.4.f Sketch the graph of simple cubic polynomial functions $(f(x)=ax^3+d)$ and radical functions $(f(x)=\sqrt{x-c}+d)$.	

HS Priority G1: LOGICAL ARGUMENTS AND PROOFS

(Geometry and Geometric Measurement)

Students use inductive reasoning to arrive at conjectures and use deductive reasoning to establish the truth of conjectures or to reject them on the basis of counterexamples. Students distinguish between inductive reasoning and deductive reasoning, and use both in Euclidean geometry. Students extend their understanding of axiomatic systems to include a basic comparison of Euclidean geometry and spherical geometry.

Performance Expectations	Explanatory Comments and Examples
Students are expected to:	
G.1.a Illustrate undefined terms, defined terms, postulates, and theorems in an axiomatic system using examples from Euclidean geometry.	
G.1.b Form conjectures about geometric relationships based on explorations (with and without technology) and inductive reasoning.	Use geometric software to conjecture that an angle inscribed in a circle has a degree measure that is half the degree measure of the intercepted arc.
G.1.c Use deductive reasoning to prove that a valid mathematical statement is true using two-column, paragraph, and flow-chart formats.	
G.1.d Identify errors or gaps in flawed mathematical reasoning; develop counterexamples to refute invalid statements.	
G.1.e Determine the validity of a proposition and its negation, converse, inverse, and contrapositive as they arise in logical reasoning.	

G.1.f Compare and contrast Euclidean geometry with basic spherical geometry.	Examples: • *A postulate of Euclidean geometry is that two points are sufficient to determine a line. What is the analog of this postulate for spherical geometry? Is the analog true? Justify your answer.* • *Use given definitions in spherical geometry to write basic postulates and compare postulates in these two geometric systems* • *Identify postulates or theorems in Euclidean geometry that may not extend to spherical geometry.*

High School Priority G2: ATTRIBUTES AND PROPERTIES OF TWO- AND THREE-DIMENSIONAL FIGURES

(Geometry and Geometric Measurement)

Students demonstrate an understanding of properties that can be used to characterize geometric figures in a plane and in space. They demonstrate a facility with Euclidean geometry's postulates and theorems, and they use them to validate properties. Students use these properties to solve problems in mathematical and real-world contexts.

Performance Expectations	Explanatory Comments and Examples
Students are expected to:	
G.2.a Identify and apply conditions that are sufficient to guarantee congruence and similarity in triangles.	Recognize that congruence is a special case of similarity.
G.2.b Use conditions for congruence and similarity of triangles, along with theorems about angles, to prove conjectures about • triangle congruence, • triangle similarity, and • properties of triangles.	
G.2.c Construct special lines associated with a triangle, including perpendicular bisectors, medians, angle bisectors, and altitudes; verify properties associated with these special lines.	
G.2.d Make conjectures and prove basic properties of quadrilaterals and other polygons.	This includes properties of angles, diagonals, symmetry, and angle sums. Example: *Determine the measure of interior and exterior angles of regular polygons and write rules that describe the relationship between the*

		number of sides and the angle measures.
G.2.e	Prove and apply the Pythagorean theorem and its converse.	
G.2.f	Use the Pythagorean theorem and similarity to establish basic trigonometric ratios of sine, cosine, and tangent.	
G.2.g	Verify and apply properties of angles, arcs, chords, secants, and tangents of circles.	Examples: • *Prove that a triangle inscribed on the diameter of a circle is a right triangle.* • *Prove that if a radius of a circle is perpendicular to a chord of a circle, then it bisects the chord.*
G.2.h	Perform straightedge-and-compass, paper-folding, and software constructions to make and test conjectures about geometric properties and relationships; justify the steps in a construction using the theorems and postulates of Euclidean geometry.	Examples: • *Construct the perpendicular bisector of a line segment. Give the postulate or theorem underlying each step.* • *Construct a circle through three non-collinear points. Give the postulate or theorem underlying each step.* • *Given a line, and a point not on the line, construct a line through the point and parallel to the given line. Give the postulate or theorem underlying each step.*
G.2.i	Describe regular and non-regular polyhedra, including the shapes of faces and the relationships between the number of faces, edges, and vertices.	"Describing" might include: • characterizing basic polyhedra, such as pyramids, parallelepipeds, and tetrahedra; • specifying the number of faces and their shapes; or • analyzing polyhedra by sketching a corresponding net.
G.2.j	Analyze cross sections formed by the intersection of planes and three-dimensional figures and identify the resulting shapes.	Describe sections obtained by cutting in different directions and sketch views from various positions. Examples:

	Describe all the possible cross sections of a cube cut by a plane not parallel to a face.
G.2.k Establish sufficient conditions for the congruence of polygons, circles, and arcs by extending the concepts of triangle congruence; determine the congruence of two polygons given information about the polygons.	Examples: • *If two quadrilaterals have AAAS measures in common, can congruency be determined? If not, provide a counter example.* • *Demonstrate why SSSSS is not a sufficient condition to determine a unique pentagon.*

High School Priority G3: GEOMETRY IN THE COORDINATE PLANE

(Geometry and Geometric Measurement)

Students use the coordinate plane to represent mathematical and contextual situations. They apply algebraic methods to interpret, represent, and verify geometric relationships. Students develop an understanding of the connection between geometry and algebra by studying geometric properties and attributes that can be described by location on a coordinate plane.

Performance Expectations	Explanatory Comments and Examples
Students are expected to:	
G.3.a Determine the algebraic equation of a line that is described geometrically in a coordinate plane.	Examples: • *Write an equation for a line given two points.* • *Write an equation for a line through a point and parallel to a given line.* • *Write an equation for the perpendicular bisector of a given line segment.*
G.3.b Determine the coordinates of a point that is described geometrically in a coordinate plane.	Examples: • *Determine the coordinates for the mid-point of a given line segment.* • *Given three coordinates of a parallelogram, determine all possible coordinates for the fourth point.*

G.3.c	Find the equation of a circle that is described geometrically in a coordinate plane; given equations for a circle and a line, determine the coordinates of their intersection(s).	Examples: • *Write an equation for a circle given coordinates of a center point and a radius.* • *Write an equation for a circle given a line segment on a coordinate plane that serves as a diameter.* • *Write an equation for a circle given coordinates of a center point and an equation of a line tangent to the circle.*
G.3.d	Use coordinate geometry to classify two-dimensional figures, solve problems, and verify properties of polygons and circles.	Examples: • *Given four vertices of a quadrilateral, verify that the quadrilateral is a rhombus.* • *Given coordinates of vertices of a polygon, draw the polygon, name it as precisely as possible, and justify your choice using algebra.*

High School Priority G4: GEOMETRIC TRANSFORMATIONS
(Geometry and Geometric Measurement)
Students study the attributes of geometric figures under rigid transformations and dilations. They study techniques for establishing congruence and similarity in relation to transformations and dilations.

Performance Expectations	Explanatory Comments and Examples
Students are expected to:	
G.4.a Sketch translations, reflections, rotations, dilations, and compositions of these transformations for a given two-dimensional figure in a coordinate plane; describe the rule(s) for performing translations and reflections about lines, including lines parallel to the coordinate axes and the line $y=x$.	
G.4.b Make and verify conjectures about • the congruence and similarity of figures under various transformations or composition of transformations, • resultant compositions and inverses of transformations, and • the commutativity and associativity of transformations.	Examples: • *Identify those transformations and compositions of transformations that preserve congruence.* • *Verify by counterexample that a composition of line reflections is not commutative.* • *Verify that composition of rotations with the same center is commutative.* • *Show by example that a rotation can be done as the composition of two reflections over intersecting lines.* • *Find the inverses of a given transformation, i.e., doing and undoing.*

G.4.c Given two congruent figures in a plane, describe a composition of translations, reflections, and rotations that superimposes one figure on the other.	
G.4.d Analyze figures in terms of their symmetries—point, line, rotation—using reflections, rotations, translations, and their compositions.	

High School Priority G5: GEOMETRY AND MEASUREMENT IN THE PHYSICAL WORLD
(Geometry and Geometric Measurement)
Students derive the formulas for perimeter, area, surface area, and volume of two- and three-dimensional figures. They specify precision in measurement and analyze the relative effect of changes in one measurement on another.

Performance Expectations	Explanatory Comments and Examples
Students are expected to:	
G.5.a Derive, verify, and apply formulas for the area and perimeter of two-dimensional figures, and arc length and area of a sector of a circle; find areas and perimeters of irregular two-dimensional figures.	In grades 9–12, the focus is on deriving and verifying formulas and applying formulas in advanced problems and complex figures.
G.5.b Apply formulas for the surface area and volume of three-dimensional figures, including right and oblique prisms, pyramids, cylinders, cones, spheres, and irregular three-dimensional figures.	
G.5.c Predict and verify the effect of changing one, two, or three linear dimensions on perimeter, area, volume, or surface area of two- and three-dimensional figures.	The emphasis in high school should be on verifying the relationships between line, area, and volume measurements and making predictions using algebraic methods.
G.5.d Provide examples of situations in which different degrees of precision in measurement are required, explain the reason why a different degree of precision is sometimes called for, and apply estimation strategies to obtain reasonable measurements with appropriate precision for a given purpose.	Examples: • *When measuring for carpeting, overestimates are better than underestimates.* • *When measuring cross-country distances, miles are better than inches.* • *Precision ball-bearings require a high degree of precision.*

G.5.e Convert from one unit of measure to another in expressions involving length, area, volume, and derived units of measure, such as speed and density; solve problems involving conversions within or between measurement systems.	This performance expectation is intended to build an understanding of unit conversions as rates or scale factors: • when making conversions, there are corresponding inverse rates (for example 1.6 km per mile or 1/1.6 miles per km), • when making conversions know which rate is appropriate, and • when making conversions, know when to multiply and when to divide or multiply by the reciprocal of a conversion factor.

High School Priority D1: UNIVARIATE AND BIVARIATE DATA AND DISTRIBUTIONS

(Data, Statistics, and Probability)

Students select mathematical models and graphical displays to represent, describe, and compare data sets using technology as appropriate. They evaluate the relationship between two variables, and make predictions using those relationships. Students make conjectures and generalizations based on data, taking into account the variability in the data. They pay explicit attention to common misconceptions and misrepresentations, and understand the limitation of the interpretation of empirical results.

Performance Expectations	Explanatory Comments and Examples
Students are expected to:	
D.1.a Determine appropriate measure(s) of center (such as mean, median, and mode) and variability (such as range, interquartile range, and standard deviation) to describe a data set and justify their choice(s).	Example: *Is mean, median, or mode the best measure of central tendency to describe typical home prices in a specific area? Why? How might the other measures be misleading?*
D.1.b Evaluate the appropriateness of summary statistics used by others and identify misleading uses of these statistics.	Summary statistics include mean, median, mode, range, interquartile range, and standard deviation.
D.1.c Determine the effect of outliers on summary statistics.	
D.1.d Evaluate the effect of linear transformations of a data set (univariate) on summary statistics.	Examples: *What is the effect of increasing everyone's wage by $200?* *What is the effect of doubling everyone's wage?*
D.1.e Determine if a linear pattern is apparent in a scatterplot (bivariate data), approximate a trend line or use technology to determine a least-squares regression line, interpret slope and intercept of the trend line in the context of the data, use the line to make predictions, and judge the reasonableness of the predictions.	Example: *The equation of the line of best fit for a given data set relating the variables A and B is* $B = -5A + 95$. *What do the -5 and 95 represent in this context?*

D.1.f Determine if a scatterplot (bivariate data) demonstrates a recognizable nonlinear relationship, use technology to approximate the exponential or polynomial curve of best fit, use the best fit equation to make predictions, and judge the reasonableness of the predictions.	
D.1.g Determine if the correlation of a given scatterplot is strong or weak and whether it is positive or negative.	
D.1.h Compare and draw conclusions about two univariate data sets using graphic displays of their distributions.	
D.1.i Use the defining characteristics of a normal distribution to identify and justify common examples that are reasonably modeled with normal distribution and those that are not.	
D.1.j Apply the empirical rule to approximate the percentage of the population meeting certain criteria in a normal distribution.	
D.1.k Determine possible causes and effects of skewed and clustered distributions, including outliers.	
D.1.l Determine whether arguments based on data confuse correlation with causation and, when they do, formulate valid inferences and conclusions based on the data.	
D.1.m Evaluate the reasonableness of and make judgments about claims, reports, studies, and conclusions.	
D.1.n Use examples to illustrate how a given set of data can be used to support different points of view.	

High School Priority D2: EXPERIMENTS AND STUDIES

(Data, Statistics, and Probability)

Students formulate questions that can be answered by collecting relevant data, analyzing the data, and interpreting the results. Students recognize that minimizing variability is key to the design of experiments and studies, including data collection, and students learn to identify potential sources of bias.

Performance Expectations	Explanatory Comments and Examples
Students are expected to:	
D.2.a Determine possible sources of bias in questions and data collection methods and/or samples, and describe how these can impact the accuracy of the results of a particular experiment or study.	
D.2.b Describe the nature and purpose of sample surveys, experiments, and observational studies, relating each to the types of research questions they are best suited to address.	Example: *Explain why observational studies generally do not lead to good cause and effect statements.*
D.2.c Select an appropriate data collection method (e.g., survey, experiment, or observation) for a given research question, and, conversely, identify specific research questions that could be addressed by a given data collection method.	
D.2.d Critique various methods of experimental design, data collection, and data presentation used to investigate real-world problems, including those reported in studies published in the media.	
D.2.e Describe and evaluate methods to select a simple random sample, and explain the rationale for using randomness in research designs.	
D.2.f Distinguish between random sampling in surveys and random assignments of treatments in experiments.	

D.2.g	Simulate the collection of data to carry out an experiment or use reliable sample data to make conjectures, draw conclusions, and/or answer a research question.	

High School Priority D3: PROBABILITY, RELATIVE FREQUENCY, AND UNCERTAINTY

(Data, Statistics, and Probability)

Students recognize that probability is the measure of the likelihood of an outcome in uncertain circumstances. They distinguish between experimental and theoretical probability to investigate and make decisions about practical situations involving uncertainty.

Performance Expectations	Explanatory Comments and Examples
Students are expected to:	
D.3.a Establish the dependence or independence of two events, determine the subset of a sample space for specific events, and use appropriate operations, including finding complements, to calculate the probability of specific independent, compound, and conditional events in finite sample spaces.	Examples: *What is the probability of getting exactly two heads when tossing a fair coin three times?* *What is the probability of getting at least two heads in three tosses of a fair coin?*
D.3.b Describe the relationship between theoretical probability and empirical frequency of events from simulations (with or without technology) in mathematical and real-world settings, and use simulations to estimate the probabilities of events where theoretical values are difficult or impossible to compute.	
D.3.c Apply the fundamental counting principle and the ideas of order and replacement to compute permutations or combinations, and use the results to compute the probability of compound events.	
D.3.d Use experimental and theoretical probability to investigate, represent, and make decisions about practical situations involving uncertainty.	
D.3.e Use probabilities to compute, interpret, and recognize common misconceptions about odds and risk.	

D.3.f	Solve problems involving geometric probability.	
D.3.g	Describe qualitatively the inverse relationship between risk and return.	Relate to simple situations, such as why teenagers pay higher car insurance premiums.

High School Priority P1: COMMUNICATION

(Communication)

Students interpret and use the symbols and language of mathematics to assimilate and communicate mathematical ideas, to reason, and to justify solutions, inferences, and conclusions. Students create and use representations to organize, record, and communicate mathematical ideas.

Performance Expectations	Explanatory Comments and Examples
Students are expected to:	
P.1.a Decode mathematical symbols and text for a given purpose.	
P.1.b Use appropriate symbols, diagrams, graphs, and vocabulary to communicate mathematical ideas and reasoning with precision and efficiency.	
P.1.c Represent mathematical and contextual situations using more than one mathematical representation or model; describe the advantages of using one or more particular representations for a given situation.	

High School Priority P2: REASONING AND PROBLEM SOLVING USING MATHEMATICS
(Reasoning and Problem Solving)
Students can describe and apply, both inductive and deductive reasoning to form conjectures, test them, and state conclusions. Students can analyze mathematical and contextual situations, recognize a problem and develop a mathematical representation of the problem and use the representation to solve the problem. Students verify that their solutions are reasonable.

Performance Expectations		Explanatory Comments and Examples
Students are expected to:		
P.2.a	Form conjectures based on exploring mathematical situations; translate conjectures into mathematical statements.	Forming and stating new conjectures is an important part of inductive thinking in mathematics. Technology (graphing calculators, dynamic software, etc.), paper folding, compass and straight-edge construction, as well as manipulation of symbols all provide experiences that can lead to conjectures.
P.2.b	Use the properties of algebra and number systems to develop a valid mathematical argument and prove a conjecture; develop a counterexample to refute an invalid statement.	
P.2.c	Select and use relevant information, determine what information is extraneous or missing, and identify and articulate implicit and explicit assumptions in the problem-solving process.	
P.2.d	Use multiple strategies for solving and/or verifying a solution to a problem.	
P.2.e	Interpret the meaning of a mathematical solution and evaluate the reasonableness of a solution to the problem.	